MAXWELL DUALITY, LORENTZ INVARIANCE, AND TOPOLOGICAL PHASE

Jonathan P. Dowling* and Colin P. Williams

Quantum Algorithms & Technologies Group
Information & Computing Technologies Research, Section 365
Jet Propulsion Laboratory, California Institute of Technology
Mail Stop 126-347, 4800 Oak Grove Drive, Pasadena, California 91109

J. D. Franson

The Johns Hopkins University
Applied Physics Laboratory
Laurel, Maryland 20723

(Received:)

We discuss the Maxwell electromagnetic duality relations between the Aharonov-Bohm, Aharonov-Casher, and He-McKellar-Wilkens topological phases, which allows a unified description of all three phenomena. We also elucidate Lorentz transformations that allow these effects to be understood in an intuitive fashion in the rest frame of the moving quantum particle. Finally, we propose a realistic set up for measuring and interpreting the He-McKellar-Wilkens phase directly in an experiment.

PACS: 03.65.B, 72.15.R, 14.80.H, 11.30.C

---

11 JUN 99, Ver. 3.2, to be submitted to *Phys. Rev. Lett.*

In 1959 Aharonov and Bohm (AB) predicted that a quantum point charge, $e = |e|$, circulating around a magnetic flux line would accumulate a quantum topological phase [1]. The flux tube can be thought as a solenoid of infinitesimal cross-sectional area or, equivalently, as a linear array of point magnetic dipoles [2], as shown in Fig. 1a. The AB phase is $\varphi_{AB} = e\Phi_M / \hbar c$, where $\Phi_M = 4\pi^2\mu$ is the magnetic flux with μ the number of dipoles per unit length. The AB effect has been confirmed by a series of elegant interference experiments, culminating in the electron holography demonstrations of Tonomura and co-workers [3]. In 1984, Aharonov and Casher (AC) predicted a reciprocal effect [4]. The AC phase accumulates on a point, quantum, magnetic dipole $\boldsymbol{m}$ as it circulates around and parallel to a straight line of charge, as depicted in Fig. 1b. The AC phase is given by $\varphi_{AC} = 4\pi m\lambda_E / \hbar c$, where λ_E is the electric charge per unit length. The AC effect has been observed experimentally with a neutron interferometer [2] and in a neutral atomic Ramsey interferometer [5].

In the 1993 and 1994, He and McKellar, and independently Wilkens, predicted the existence of a third topological phase, depicted in Fig. 1d, that is essentially the Maxwell dual of the AC effect [6]. Wilkens called this the Röntgen phase for historical reasons [7], but we will refer to it here as the He-McKellar-Wilkens (HMW) phase. In the HMW effect, a point electric dipole $\boldsymbol{d}$ accumulates a topological phase while circulating around, and parallel to, a line of magnetic charge (monopoles). To relate this to the AC effect, note that Maxwell's equations are invariant under the electric-magnetic duality transformations given by [4], $\mathbf{A}_E \to \mathbf{A}_M$, $\mathbf{E} \to \mathbf{B}$, $e \to g$, $\mathbf{d} \to \mathbf{m}$, and $\mathbf{A}_M \to -\mathbf{A}_E$, $\mathbf{B} \to -\mathbf{E}$, $g \to e$, $\mathbf{m} \to \mathbf{d}$. Here, g is a unit of north magnetic monopole charge, and $\mathbf{A}_E$ and $\mathbf{A}_M$ are electric and magnetic vector potentials, defined in the absence of electric or magnetic monopoles, respectively. Hence, we define $\mathbf{B} = \nabla \times \mathbf{A}_M$ with $\nabla \cdot \mathbf{B} \equiv 0$, or

$\mathbf{E} = \nabla \times \mathbf{A}_E$ with $\nabla \cdot \mathbf{E} \equiv 0$. Therefore, as shown by He and McKellar, any derivation of the AC effect is also a derivation of the HMW effect, via the above duality transformations [6]. By inspection, we can therefore write down the HMW phase as $\varphi_{HMW} = -4\pi d\lambda_M / \hbar c$. This result is in agreement with Wilkens's calculation, which considered the general problem of a dipole moving in an arbitrary electromagnetic field [6, 7]. Here, λ_M is the magnetic monopole charge per unit length, and the minus sign arises from the asymmetric nature of the duality transform. Interestingly enough, our duality analysis also predicts a fourth phenomenon, which is the dual of the AB effect (see Fig. 1c). Here, a quantum point magnetic monopole acquires a topological phase as it circumnavigates a line of electric dipoles (electric flux tube). Once again, any derivation of the usual Aharonov-Bohm effect is precisely one of the dual Aharonov-Bohm (DAB) effect, through the duality transform. By inspection, the DAB phase is $\varphi_{DAB} = -g\Phi_E / \hbar c$, where now $\Phi_E = 4\pi^2\delta$ is the electric flux, with δ the number of electric dipoles per unit length. (We would like to note that the DAB phase has also been discussed independently by Wilkens in unpublished public communications.)

Of course the immediate question is how to observe these dual topological phases experimentally. Both dual effects seem to require free magnetic monopoles, which have never been reliably observed. For the case of the HMW phase, Wilkens proposed employing a pierced magnetic sheet to mimic the required radial magnetic field and then utilizing a matter-wave interferometer for molecules (with a permanent dipole moment) [6]. Apart from the technical challenges involved in such an interferometry experiment with molecules, the question of whether or not this scheme will work even in principle has been debated [8]. Several other authors have proposed variations on Wilken's original theoretical exposition or his proposed experimental configuration [9], but not without controversy [10]. So far the effect has not been seen, or at least recognized, in any experiment. It is the purpose of this paper to provide a concrete physical setup in which the HMW phase could be observed and interpreted correctly. Unfortunately, the observation of the DAB effect does seem to require unphysical free magnetic monopoles.

The AB, AC, DAB, and HMW phases can be written in integral form as

$$\varphi_{AB} = \frac{e}{\hbar c}\oint \mathbf{A}_M \cdot \mathbf{dl} = e\Phi_M / \hbar c, \qquad \varphi_{AC} = \frac{1}{\hbar c}\oint [\mathbf{m}\times\mathbf{E}]\cdot\mathbf{dl} = 4\pi m\lambda_E / \hbar c, \tag{1a}$$

$$\varphi_{DAB} = -\frac{g}{\hbar c}\oint \mathbf{A}_E \cdot \mathbf{dl} = -g\Phi_M / \hbar c, \qquad \varphi_{HMW} = -\frac{1}{\hbar c}\oint [\mathbf{d}\times\mathbf{B}]\cdot\mathbf{dl} = -4\pi m\lambda_M / \hbar c, \tag{1b}$$

where $\mathbf{A}_{\mathrm{M}}$ and $\mathbf{A}_{\mathrm{E}}$ are the magnetic and dual electric vector potentials. The dual DAB and HMW phases, Eq. (1b), are obtained from Eq. (1a) by the duality transforms. All four of these phases are topological in that the result does not depend on the particle velocity or the particular circulating path taken. They are all nonclassical in the sense that there is no classical force acting on the particle as it circulates, and that the effect manifests itself in the quantum phase of the wavefunction. In addition, in the AB and the DAB effect, there is the additional non-intuitive feature that there are no physical $\boldsymbol{B}$ or $\boldsymbol{E}$ fields at the location of the moving charge—only the gauge fields $\mathbf{A}_{\mathrm{M}}$ and $\mathbf{A}_{\mathrm{E}}$ are present.

It is important to note that Hinds and co-workers have shown that the AC effect can be thought of as a motional Zeeman shift [5]. Consider a Lorentz frame K' that is co-moving with any of the circulating particles in Fig. 1. The Lorentz transforms for the electric and magnetic fields, in the small velocity limit, are then [4],

$$\mathbf{E}' \cong \mathbf{E} - \mathbf{v}\times\mathbf{B}/c \;, \qquad \mathbf{B}' \cong \mathbf{B} + \mathbf{v}\times\mathbf{E}/c \;. \tag{2}$$

Consider the AC effect, Fig. 1b, where $\boldsymbol{B} = 0$ in the lab frame, and $\mathbf{E} = 2\lambda_E \hat{\boldsymbol{\rho}} / \rho$ is the radial electric field from the line of charge. (Here, $\boldsymbol{\rho}$ and $\hat{\boldsymbol{\rho}}$ are the radial and radial-unit vectors, respectively, in cylindrical coordinates.) As the magnetic dipole circulates about the line of charge, in the co-moving frame the dipole couples to the field via the Hamiltonian, $H = -\mathbf{m} \cdot \mathbf{B}'$, giving rise to a Zeeman phase shift, $\varphi = \oint H(t)dt / \hbar$, integrated over the circulation time. This is precisely the AC phase when one Lorentz transforms back into the lab frame and performs a change of variable, $\boldsymbol{v} = \boldsymbol{dl}/dt$, where $dl = \rho d\varphi$. Our simple argument here is good to first order in v/c, whereas the more complete treatment of Hinds and colleagues is good to all orders [5]. Once again, Maxwell duality tells us immediately that the dual HMW phase is then—to all orders in v/c—a motional *Stark* effect, with $H = -\mathbf{d} \cdot \mathbf{E}'$ in the co-moving frame of the electric dipole. In the HMW lab frame, $\boldsymbol{E} = 0$, and hence $H = \mathbf{d} \cdot (\mathbf{v} \times \mathbf{B} / c)$, which is exactly the so-called Röntgen interaction [7]. We see from this analysis that the Röntgen contribution is a motional Stark shift, when viewed from the co-moving frame.

For completeness, let us also analyze the AB and DAB effects in the co-moving frame. A vector potential transforms as the vector component of a covariant four-vector as, in the small velocity limit [4], $V' \cong V - \mathbf{v} \cdot \mathbf{A} / c$, $\mathbf{A}'_{||} \cong \mathbf{A}_{||} - V\mathbf{v} / c$, and $\mathbf{A}'_{\perp} \cong \mathbf{A}_{\perp}$. Here, V is the scalar electric potential, and the parallel and perpendicular notation is with respect to the direction of motion. The general interaction Hamiltonian is $H = -eV + e\mathbf{p} \cdot \mathbf{A} / mc$, but this reduces to $-eV'$ in the co-moving frame where the momentum $\boldsymbol{p}' = 0$. Lorentz transforming to the lab frame, we get,

$$\varphi = \frac{1}{\hbar}\oint H(t)dt = -\frac{e}{\hbar}\oint V'dt = \frac{e}{\hbar c}\oint \mathbf{A} \cdot \mathbf{v}dt = \frac{e}{\hbar c}\oint \mathbf{A} \cdot \mathbf{dl} , \quad (3)$$

which is clearly the AB phase, Eq. (1a). The interpretation is that in the co-moving frame the circulating charge experiences a constant, velocity-independent voltage difference across the clockwise and counter-clockwise circulating branches of the interferometer. This electrostatic potential gives rise to the AB phase shift when integrated over the circulation time. It is easy to show in the slow velocity limit that $\boldsymbol{E}'$ and $\boldsymbol{B}'$ are still be zero in the co-moving frame. In particular, to first order in v/c, the time-independent electrostatic potential V' is such that its gradient is zero in this frame, so there is no corresponding electric field. Since $\boldsymbol{A}' = \boldsymbol{A}$, the curl of both vector potentials is zero in the small velocity approximation, and so there is no magnetic field (external to the solenoid) in either frame. Similar arguments hold for the DAB phase.

Finally, we move to our experimental proposal for observing and interpreting the HMW phase. The key is to employ the same transformation that Hinds and co-workers used to enhance the AC effect in neutral atoms [5]. The difficulty with an experimental demonstration of the AC effect with matter-wave interferometry is that it is challenging to maintain a large enough voltage on a single wire of charge to see the effect in the configuration of Fig. (2a). In Fig. (2b), one sees that the same AC phase would be obtained if the dipoles were made to follow the same path on one side of the wire, but with a superposition of up and down magnetic dipole moments within the particle instead. Once this is acknowledged, then it is simple to see that the single wire may be replaced with a parallel plate capacitor that maintains the same constant electric field along a now straight trajectory, shown in Fig. (2c). Since the electric field across the two plates of a capacitor can be made very large compared to that near an isolated wire, this enhances the AC effect so that it may be measured more easily. The superposition of magnetic moments is prepared by a exciting an ensemble of magnetic sublevels judiciously, and the measurement of the phase is made by two-pulse Ramsey interferometry [5].

By Maxwell duality—the same transformation will work for a measurement of the HMW effect in neutral atoms. In this case the plus and minus signs in Fig. (2) represent north and south magnetic charge, and the moving quantum dipole is electric. For our purpose here, this amounts to replacing the unphysical line of magnetic monopoles, Fig. (2a), with the north and south poles of an ordinary electromagnet, Fig. (2c). For the measuring of the HMW phase, this transformation does not make the experiment easier—it is what makes it possible at all. The need for unphysical magnetic monopoles has been eliminated. Finally, all that is left to do is to specify a realistic atomic state that can be prepared easily in a superposition of up and down electric dipole moments. Nature provides us with such a system in an excited hydrogen-like atom.

Consider the original HMW configuration of Wilkens, Fig. (2a), transformed into the Hinds configuration of Fig. (2c). The Hinds set up does not require spatially separated paths—merely a superposition of dipoles in one atom on one path. The excited hydrogen-like atoms have a degeneracy of states with opposite parity. Hence, the superposition of such states results in energy eigenstates without definite parity. That is, the expectation of the electric dipole moment does not vanish [11]. It is this effect that is responsible for the existence of the first-order Stark effect in hydrogenic atoms, an effect that vanishes in non-hydrogenic systems. Consider the $2s$ and $2p$ first-excited states of hydrogen, $|nlm\rangle = |2lm\rangle$, where $l \in \{0, 1\}$ and $m \in \{-1, 0, 1\}$. Applying degenerate perturbation theory, the linear Stark effect splits the degenerate $m = 0$ level into two components, with energy eigenvalues and eigenfunctions given by

$$\Delta\varepsilon_{\pm} = \pm 3 a_B e E' \ , \qquad\qquad \left|\psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\{|200\rangle \pm |210\rangle\} \ , \tag{4}$$

where E' is the electric field in the co-moving frame. The $|nlm\rangle$ eigenstates have definite parity and hence zero dipole expectation value. However, in the new basis the states $\left|\psi^{\pm}\right\rangle$ have a definite up and down dipole values. Inverting the transformation of Eq. (4), we find for the *2s* and *2p* states $|200\rangle = \left\{\left|\psi^{+}\right\rangle + \left|\psi^{-}\right\rangle\right\} / \sqrt{2}$ and $|210\rangle = \left\{\left|\psi^{+}\right\rangle - \left|\psi^{-}\right\rangle\right\} / \sqrt{2}$, respectively. This representation shows directly that these excited states are equal superpositions of up and down electric dipole moments—precisely the superposition that is required for the Hinds-transformed HMW configuration in Fig. (2c).

For our proposed experiment we can use a simple time-of-flight measurement, which is in principle the same set up as use by Hulet, Hilfer, and Kleppner [12]. They measured cavity QED changes in spontaneous decay rates of atoms in flight between parallel mirrors. For our experiment, it is perhaps easiest to think of the HMW phase as a Stark shift in the co-moving frame of the atom in order to visualize how the proposal works. Suppose we have a source of metastable hydrogen atoms in the *2s* state, as shown in Fig. 3. Such sources are easily made by pulsed electron bombardment of a beam of hydrogen molecules [13–16]. The resultant metastable hydrogen atoms have an exceedingly long lifetime of about 0.14 s since the transition to the ground state is dipole forbidden [18]. So, at typical atomic beam velocities of 10^6 cm/s, the lifetime is effectively infinite during passage through, say, a 1.0 cm magnetic field region. As the atom propagates the metastable *2s* state will, in the co-moving frame, experience a field E', as per Eq. (2), and will be Stark shifted by the amount given in Eq. (4). Integrating over the time of flight between the magnet poles and transforming into the lab frame, via Eq. (2), the HMW phase and the time-dependent wavefunction become [18],

$$\varphi_{HMW} = 3a_B eBL / \hbar c \; = 3a_B eBvt_0 / \hbar c \ , \tag{5a}$$

$$|\Psi(t)\rangle = |200\rangle \cos(\varphi_{HMW}\, t / t_0) + i|210\rangle \sin(\varphi_{HMW}\, t / t_0) \ . \tag{5b}$$

Here, a_B is the hydrogenic Bohr radius, L the distance flown between the magnetic poles, and t_0 the corresponding time of flight. If B is measured in Gauss and L in centimeters, then Eq. (5a) can be written $\varphi_{HMW} = 0.24\,BL$. The excited atomic wavefunction mixes states of different parity, since orbital angular momentum is not conserved in an electric field [18]. However, the *2p* state decays rapidly to the ground state by a dipole transition and has a very short lifetime of only $\tau = 1.6 \times 10^{-9}$ s. Therefore, once in the *2p* state the atom will decay relatively instantaneously into *1s* over a distance of about 16 mm; long before reaching the detector. As the wavefunction oscillates between *2s* and *2p*, as per Eq. (5b), the HMW phase shift acts as beam dump that converts metastable *2s* atoms into ground state *1s* hydrogen, at a rate depending on the phase-dependent oscillation period, $T = \pi t_0 / \varphi_{HMW}$. This electric-field-induced decay or "Stark quenching" of metastable, excited, hydrogen-like atoms is a previously known effect that has already been measured in the lab. It has been seen both with applied external electric fields [14] and with motionally induced Röntgen electric fields [15–17]. In the latter case, the effect was used as a magnetic-field dependent beam polarizer in 1952 by Lamb and Retherford [15] and more recently by Robert, *et al.*, as a velocity selector [16]. In fact, the earliest observation of this motional Stark effect was probably made in 1916 by Wien [17]. What is new here is the interpretation of the phenomenon in terms of the HMW phase.

The dumping of *2s* to *1s* states is most efficient when the period of the *2s* to *2p* oscillation is near the *2p* decay rate [16]. This resonance occurs at the motional Stark-induced level crossing between the *2s* and *2p* states [18]. For the rest of this work we will assume a beam velocity of $\upsilon = 10^6$ cm/s. In this case, the resonance condition requires a rather strong field of $B = 8.12\times 10^3$ G, which will provide almost complete conversion. However, at smaller fields we may take the full Stark-quenched, radiation-rate equations [14], and integrate them over solid angle to compute the total field-induce atomic decay rate γ. To first order in B and the fine structure constant α, this rate can be written as,

$$\gamma = \frac{3^{10}}{2^8}\frac{1}{\alpha^3}\frac{a_B^3}{\hbar}\left(\frac{\upsilon B}{c}\right)^2 = \frac{3^8}{2^8}\frac{1}{\alpha^4}\frac{a_B}{ct_0^2}\varphi_{WR}^2 , \tag{6}$$

where the HMW phase φ_{HMW} is given by Eq. (5a). If we take B in units of Gauss, this can be written conveniently as $\gamma = 92.6\, B^2$, emphasizing the quadratic dependence on the magnetic field. Taking an exponential decay of the form $N = N_0 \exp(-\gamma t_0)$ with $L = 1.0$ cm, then $t_0 = 10^{-6}$ s and the initial metastable flux will decay to $1/e$ of its original value for a field on the order of $B \cong 100$ G. These numbers are in good agreement with the velocity-selection experiments of J. Robert, *et al.* [16]. A magnetic field of this particular strength induces a HMW phase of around $\varphi_{jHMW} \cong 8\pi$.

In summary, we have considered the reciprocity and Maxwell duality transformations among the four topological phases found in the Aharonov-Bohm (AB), Aharonov-Casher (AC), dual Aharonov-Bohm (DAB), and He-McKellar-Wilkens (HMW) effects. In particular, the DAB and HMW effects are derived trivially from the AB and AC effects via Maxwell duality. In addition, we have looked at the simplification that comes from Lorentz boosting into the co-moving frame of the quantum particle. In this frame the AB and DAB fields appear as if induced by a constant electrostatic or magnetostatic potential differential, and the AC and HMW effects are interpreted as motional Zeeman and Stark effects, respectively. Finally, we propose a specific transformation of the HMW configuration into an ordinary dipole electromagnet set up, that allows for the experimental observation and interpretation of the HMW phase. This phase is seen in the excited states of hydrogen-like atoms, via the first order Stark shift in the co-moving frame. Such motional Stark shifts have been seen already experimentally, but have not hitherto been interpreted in terms of the HMW phase [15–17].

ACKNOWLEDGEMENTS

We would like to acknowledge interesting and useful discussions with Xiao-Gang He, Ed Hinds, and Martin Wilkens. This work was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration Additional support was provided by the Office of Naval Research.

REFERENCES

* Electronic address: Jonathan.P.Dowling@jpl.nasa.gov

1. Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).

2. A. Cimmino, et al., Phys. Rev. Lett. **63**, 380 (1989).

3. M. Peshkin and A. Tonomura, *The Aharonov-Bohm Effect* (Springer-Verlag, Berlin, 1989).

4. J. D. Jackson, Classical Electrodynamics, 2nd ed., (John Wiley & Sons, New York, 1975) Ch. 11.

5. K. Sangster *et al.,* Phys. Rev. Lett. **71**, 3641 (1993);K. Sangster *et al.,* Phys. Rev. A **51**, 1776 (1995).

6. X. G. HE, B. H. J. McKellar, Phys. Rev. A **47**, 3424 (1993); M. Wilkens, Phys. Rev. Lett. **72**, 5 (1994).

7. M. Wilkens, Phys. Rev. A **48**, 570 (1994); U. Leonhart and M. Wilkens, Europhys. Lett. **42**, 365 (1998).

8. M. S. Chapman, *et al.,* Phys. Rev. Lett. **74**, 4783, (1995); G. Spavieri, Phys. Rev. Lett. **81**, 1533 (1998); M. Wilkens, Phys. Rev. Lett. **81**, 1534 (1998).

9. H. Liu, X. Y. Huang, and S. W. Qian, Chinese Phys. Lett. **12**, 327 (1995); J. Yi, G. S. Jeon, and M. Y. Choi, Phys. Rev. B **52**, 7838 (1995); H. Q. Wei, R. S. Han, and X. Q. Wei, Phys. Rev. Lett. **75**, 2071 (1995); C. C. Chen, Phys. Rev. A **51**, 2611 (1995).

10. C. R. Hagen, Phys. Rev. Lett. 77, 1656 (1996); H. Q. Wei, X. Q. Wei, R. S. Han, Phys. Rev. Lett. 77, 1657 (1996).

11. E. Merzbacher, *Quantum Mechanics,* 2nd ed. (John Wiley & Sons, New York, 1970) Ch. 17, Sec. 6.

12. R. G. Hulet, E. S. Hilfer, D. Kleppner, Phys. Rev. Lett. **55**, 2137 (1985).

13. F. Biraben, *et al.,* Rev. Sci. Instrum. **61**, 1468 (1990).

14. G. W. F. Drake, *et al.,* Phys. Rev. A **46**, 113 (1992).

15. W. E. Lamb Jr. and R. C. Retherford, Phys. Rev. **86**, 1014 (1952).

16. J. Robert, *et al.,* Europhys. Lett. **9**, 651 (1989).

17. W. Wien, Ann. der Phys., **49**, 842 (1916); E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra,* (Cambridge University Press, Cambridge, 1959), Ch. 17, p. 402.

18. I. I. Sobelman, *Atomic Spectra and Radiative Transitions,* 2nd ed. (Springer-Verlag, Berlin, 1992), Sec. 7.2.

FIGURE CAPTIONS

Fig. 1 We indicate the geometries of the Aharonov-Bohm (AB), Aharonov-Casher (AC), dual AB (DAB), and He-McKellar-Wilkens (HMW) topological phase configurations in (a), (b), (c), and (d). In the AB effect (a), an electric charge e circulates around a line of magnetic dipoles $\boldsymbol{m}$. In the reciprocal AC effect (b), a magnetic dipole circulates around a line of electric charge. The Maxwell dual of these two effects are shown in (c) and (d). In the DAB effect (c), a magnetic charge (magnetic monopole) g circulates around a line of electric dipoles $\boldsymbol{d}$. Finally, in the HMW effect, an electric dipole circulates around a line of magnetic monopoles. In all cases the topological phase accumulated by the circulating quantum particle is independent of the path, and there is no classical force on the particle.

Fig. 2 The transformation used by Hinds and co-workers to implement an experimental observation of the AC effect in neutral atoms. Starting with the original AC configuration (a), one realizes that the same interference effect occurs if the particle circulates only counter clockwise, but in an superposition of up and down dipole moments, (b). Since the field is constant along the path in (b), one can replace the line of charge with a parallel plate configuration (c). This same transformation then applies to the HMW effect, where the dipole is now electric and the line of charge is magnetic. This transforms an unphysical line of magnetic monopoles into a realizable North-South pole geometry found in ordinary electromagnets.

Fig. 3 Proposed time-of-flight experiment to measure the He-McKellar-Wilkens phase. Metastable hydrogen atoms are generated in the atom source, and then enter the magnetic field region where the HMW phase couples the metastable *2s* to the rapidly decaying *2p,* as per Eq. (5). The excited state experiences a HMW phase shift that is proportional to magnetic field B, producing a B-dependent dumping of the metastable *2s* into the ground state *1s*. The conversion rate is determined by the counting rate of either *2s* or *1s* at the final detector and is proportional to B^2.

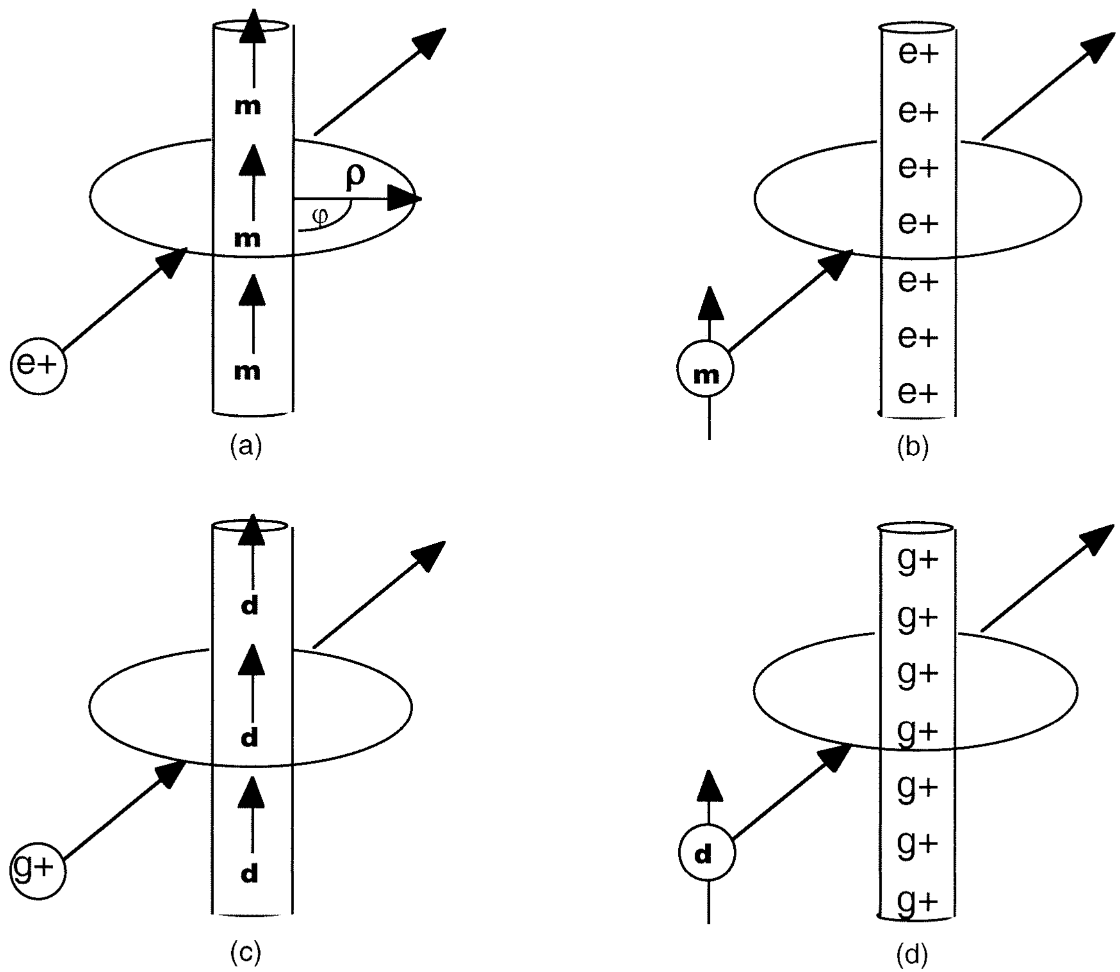

Fig. 1

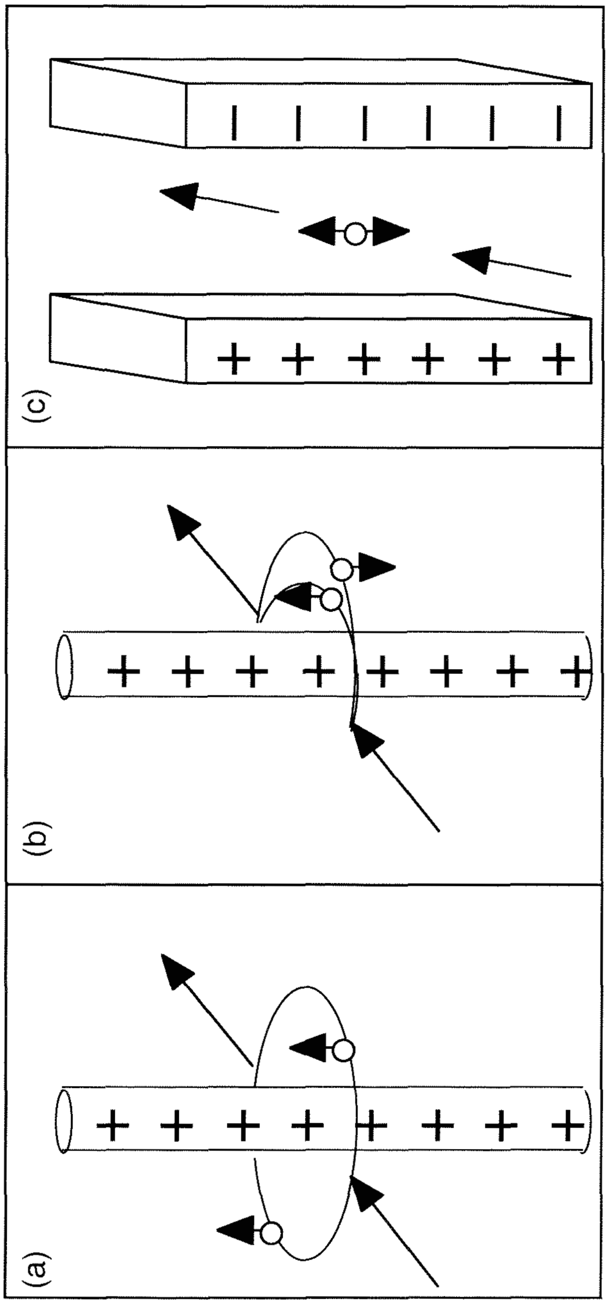

Fig. 2

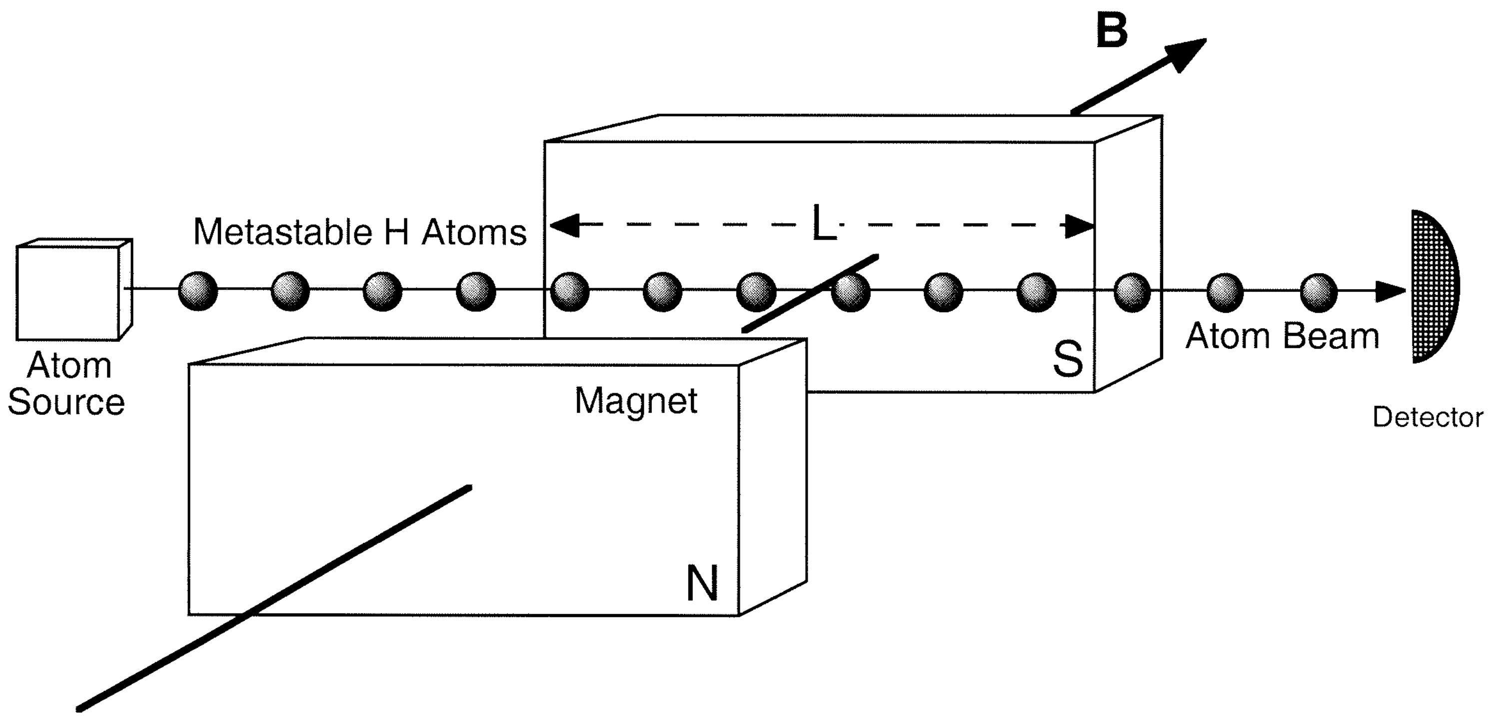
B
Metastable H Atoms
L
Atom
Source
Magnet
S
N
Atom Beam
Detector

Fig. 3